HT2013-17219

ANALYTICAL MODELING OF MIST CONDENSATION BY NATURAL CONVECTION OVER INCLINED FLAT SURFACES

Mohammad Ali Fayazbakhsh
Laboratory for Alternative Energy Conversion
(LAEC), School of Engineering Science, Simon
Fraser University
Surrey, BC, V3T 0A3, Canada

Majid Bahrami
Laboratory for Alternative Energy Conversion
(LAEC), School of Engineering Science, Simon
Fraser University
Surrey, BC, V3T 0A3, Canada

ABSTRACT

Dew formation on a transparent surface creates a pattern that can cause blurred view over it. This fogging phenomenon should be avoided in many applications. Mist condensation of water on a cold surface exposed to humid air is studied in this work. In order to analyze the misting process, the fluid flow and heat transfer of humid air as well as the heat transfer across the solid surface are considered. A dew formation model is used to predict the shape and size of the droplets. Analytical models have been proposed to solve the heat and mass transfer for the simple arrangement of a vertical flat surface. The analytical model is then combined with the dew formation model to introduce an analytical model for mist condensation over vertical and inclined surfaces. Due to the proposed method, complex numerical calculations can be avoided for solving the heat and mass transfer equations.

INTRODUCTION

The exposure of a smooth cold surface to warmer humid air can trigger the condensation of the vapor content over the surface, if the surface is colder than the dew point temperature. The condensation starts by forming mist over the surface. By prolonged exposure to humid air, the condensation may continue in the form of dropwise condensation while the droplets grow in size and coalesce with each other to generate big drops of water. In this regime, the weight of the drops will overcome the surface tension forces in scale, which causes them to become unstable and flow one after another. Merte and Yamali [1] studied the profile and departure size of condensation drops on vertical surfaces. They introduced an analytical method for estimating the stability threshold of liquid drops over vertical surfaces. For higher temperature differences, more air flow, or more humid air, the condensation rate may increase even more and the process will continue in the film condensation regime.

In many applications, the condensation happens in the mist regime only. The mist formation, also called fogging, causes the light rays to scatter due to the different refractive index of water compared to air, as well as the size and shape of the droplets. In many applications with transparent surfaces, the generation of mist creates a blurred view across the surface. This blur is sometimes associated with safety concerns as in the case of a vehicle, train, or airplane windshield. In some applications the blur of a glass surface can harm the functioning of an instrument or a gauge. In such applications, the fogging over the surface should be avoided.

Fluid flow and heat transfer of humid air as well as heat transfer across the solid surface are important for understanding and modeling of the condensation process. A dew formation model can use these results from heat and mass transfer analyses in order to predict the shape and size of the droplets. Studies show that the heat and mass transfer governing equations can be solved by conjugate numerical simulation of fluid flow and heat transfer in both fluid and solid domains. As an example, Aroussi *et al*. [2] numerically simulated the air flow and heat transfer in a vehicle demisting system. They considered the turbulent three-dimensional air flow in a simplified windshield demisting system and successfully validated their results with experiments.

Croce *et al*. [3] developed a method for solving the conjugate flow and heat transfer equations in a fluid/solid continuum pair. They introduced an iterative method for updating the temperature at the fluid/solid interface until the temperature and its corresponding heat flux matched on both fluid and solid sides. The method, coupled with a droplet formation model for the surface condensation was applied to applications such as windshield defogging [4], refrigerated cabinet doors [5], finned dehumidifiers [6], and glass fogging [7]. In all of the applications, they considered the water

layer as a collection of closely packed droplets, which left a portion of dry area in between droplets.

Any successful solution of the mist condensation requires a suitable dew formation model. Beysens [8,9] studied the formation, nucleation, and growth of dew droplets over solid surfaces. It was shown that at an early stage of mist condensation, typically at a surface coverage of more than 30%, the evolution pattern remains "self-similar" in time for many types of liquids. This self-similar regime is reached due to repeating interactions between droplets by coalescence. Since the merging of droplets is a volume-conserving process, larger droplets are produced and therefore some surface area is dried up. Ongoing condensation increases the droplet radii while on the same time increasing the number of coalescences and creating dry surface among the droplets. Beysens [8] argued that these competing trends produce a constant wet-to-total area ratio of 55%. This ratio is universal and holds for mist condensation of many fluids [8].

As such, a clear understanding of the condensation process is necessary to provide engineering insight on the chances of fogging generation and condensation rate. Developing an accurate and easy-to-use model that can predict the misting process will be beneficial for a variety of engineering applications. In this study, analytical models are proposed to solve the natural convective heat and transfer for the simple arrangement of a flat inclined surface. The analytical model is then combined with a dew formation model to introduce an analytical model for mist condensation on flat surfaces. Using the proposed model, conjugate and tedious numerical calculations can be avoided for solving the heat and mass transfer equations.

MODEL DEVELOPMENT

Laminar flow and quasi steady state assumptions have been considered to generate a model for condensation of droplets in the mist regime. Figure 1 shows the schematic of the problem with the associated coordinate axes.
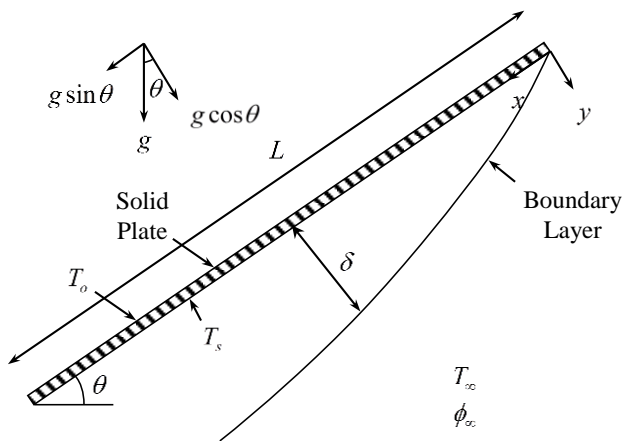

Figure 1 - Schematic of the problem

The most important assumption in the development of a model for mist condensation in this study is that the average size of droplets on the surface is much smaller than the scale of the associated flow and heat transfer phenomena. The thickness of the boundary layer presents a reasonable scale for the natural convective flow over the surface. The hydrodynamic boundary layer thickness for the natural convection case is analytically estimated to be [10]

$$\delta_{natural} = 3.936\left(\frac{0.952 + \text{Pr}}{\text{Pr}^2}\right)^{1/4}\frac{x}{\text{Gr}_x^{1/4}} \tag{1}$$

where Pr is the Prandtl number, x is the vertical position along the surface as shown in Fig. 1, and Gr is the Grashof number. Using the Grashof number definition, Eq. (1) can be rewritten as

$$\delta_{natural} = 3.936\left[\frac{(0.952 + \text{Pr})\alpha^2}{g\beta(T_\infty - T_s)}\right]^{1/4} x^{1/4} \tag{2}$$

where α is the thermal diffusivity of the liquid, g is the gravitational acceleration, β is the volumetric thermal expansion coefficient, and T_∞ and T_s are the ambient fluid and the surface temperatures, respectively. As an example, assuming air at $20°C$ as the ambient fluid, a temperature difference of $20°C$, and a vertical position of $0.1\,m$, the hydrodynamic boundary layer thickness is estimated to be about $10\,mm$. Since the air Prandtl number is almost 0.72 we can conclude that the thickness of the thermal boundary layer is also on the same order of magnitude.

On the other hand, for a case of mist condensation by humid air over a flat surface, the generated fog consists of droplets of micron-scale average diameter. Numerical simulations of D'Agaro *et al*. [5] confirm that the order of magnitude of the average droplet diameter is about $1\,\mu m$. Beysens [8] also argued that during the growth of isolated droplets, vapor diffuses to the surface in a thin layer where the flow of fluid is negligible. Hence, it can be reasonably assumed that the droplets are smaller than the boundary layer thickness so much that they do not interfere with the flow domain.

After prolonged exposure of the surface to humid ambient fluid (steady-state condition), condensation continues to occur until the process regime changes from mist condensation to dropwise condensation when relatively larger drops of liquid flow over the surface due to the gravity force overcoming the surface tension for holding the droplets in place. After even more exposure, film condensation occurs. In this study, we are focusing on the mist condensation, and dropwise and film condensation regimes are not considered. Thus, the droplets are assumed stationary and tiny.

Considering a mixture of gas and vapor such as humid air, the conservation equations of heat and mass transfer for natural convective flow over an inclined flat surface can be solved by considering certain assumptions. Bejan [11] has elaborated on the analytical solution of this type of problem and has studied the trends of the boundary layer growth in such flows. Lin and

Wu [12] solved the combined heat and mass transfer problem analytically. When gradients of both temperature and species concentration exist, the natural convective flow may be mainly instigated by the thermal convection, species convection, or a combination of both.

In many applications such as humid ambient air in contact with a flat vertical surface, the mole fraction of water vapor is small in the boundary layer and the concentration gradient of vapor has very little contribution to the buoyancy forces acting on the fluid. Thus, we can assume the mass transfer over the flat surface to be heat-transfer-driven. We also assume that all fluid properties except the vapor saturation pressures have negligible variation in the boundary layer. For the laminar flow of humid air over a vertical flat plate, the thermally-induced Sherwood number is correlated from the following equation within a maximum error of less than 1.2% for fluids with $Pr = 0.7$ and the Schmidt number ranging from 0.22 to 2.1 [12]:

$$Sh_x = \left(0.6532 Sc^{1/3} - 0.1955\right) Ra_x^{1/4} \qquad (3)$$

where Sh_x is the Sherwood number, Sc is the Schmidt number, and Ra_x is the Rayleigh number defined as

$$Ra_x = \frac{g\beta}{\nu\alpha} x^3 \left(T_\infty - T_s\right) \qquad (4)$$

where ν is the kinematic viscosity of the fluid. Bejan [11] reported that for inclination angles changing from vertical plate down to an inclination angle of $\theta = 30°$, the solution of natural convection over a vertical flat plate can be generalized by replacing the gravitational acceleration g by $g\sin\theta$ where θ is the inclination angle. Hence, we can use the vertical plate solutions to find the local boundary layer thickness and the Sherwood number. Thus, keeping the definition of the Rayleigh number based on the direction along the surface, the Sherwood number on an inclined surface will be

$$Sh_x = \left(0.6532 Sc^{1/3} - 0.1955\right) Ra_x^{1/4} \sin^{1/4}\theta \qquad (5)$$

Based on the definition of the Sherwood number, the local mass entrainment of vapor from the mixture onto the surface can thus be written as

$$\dot{m}'' = \frac{Sh_x D}{x R T_m}\left(\phi_\infty p_\infty - p_s\right) \qquad (6)$$

where D is the binary mixture mass diffusivity, R is the specific gas constant, T_m is the mean temperature, ϕ_∞ is the ambient relative humidity, and p_∞ and p_s are the vapor saturation pressures at the ambient and surface temperatures, respectively.

Equations (5) and (6) are used to calculate the amount of vapor condensation rate at a local position x along the surface. A significant portion of the heat transfer between the surface and the ambient fluid is due to the latent heat released to the fluid due to condensation. However, the difference between the surface and ambient temperatures also causes sensible heat transfer. Since the thickness of the condensate layer is small, it can be assumed that the temperature is constant across the

droplets and the ambient fluid mixture is in contact with a flat surface at temperature T_s. The corresponding Nusselt number of this heat transfer process can be found from [12]

$$Nu_x = 0.3861 Ra_x^{1/4} \qquad (7)$$

where Nu_x is the local Nusselt number. Again, based on the definition of the Nusselt number, the heat transfer rate can be calculated as

$$\dot{q}''_{convection} = 0.3861 \frac{Ra_x^{1/4}}{x} k_\infty \left(T_\infty - T_s\right) \qquad (8)$$

where k_∞ is the thermal conductivity of the ambient fluid.

Finally, the heat transfer across the surface layer is calculated from the conduction across the surface body, which results in

$$\dot{q}''_{surface} = \frac{k_s}{t}\left(T_s - T_o\right) \qquad (9)$$

where $\dot{q}''_{surface}$ is the conductive heat transfer across the surface, k_s is the solid surface thermal conductivity, t is the surface thickness, and T_o is the outside surface temperature.

In order to find the latent heat due to the condensation of droplets on the surface, a model for the formation of dew droplets is required. Following the study of D'Agaro et al. [5] and Beysens [8], a model is incorporated as shown in Fig. 2.
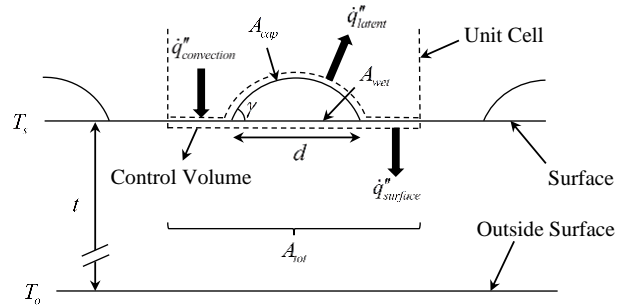

Figure 2 - Droplet model geometry

When the condensation process starts, the surface is assumed to be covered by closely packed collection of tiny droplets. Although the dimensions of the droplets are not exactly the same as each other, it can be assumed that the surface is covered by a set of spherical caps of condensate with a constant base diameter d and a contact angle γ. During this phase of mist condensation, liquid droplets form on the surface and coalesce to make larger droplets. Since this process of droplet formation and coalescence repeatedly occurs over the surface, the mist condensation phase is called the self-similar condensation regime of drop formation [6]. Because of the repetition of the drop formation processes during the self-similar regime, some areas of the surface remain dry. Beysens [8] argued that during most of the self-similar droplet condensation process, the ratio of the wet to the total surface area A_{wet}/A_{tot} remains at a constant value of 0.55 for many

fluids. In another study, De Candido *et al*. [13] also confirmed this value for condensation processes.

From Fig. 2, it can be geometrically proven that for a spherical cap, as a liquid droplet, the ratio of the cap to the total base area A_{cap}/A_{tot} is

$$\frac{A_{cap}}{A_{tot}} = \frac{2}{1+\cos\gamma}\frac{A_{wet}}{A_{tot}} \qquad (10)$$

where γ is the contact angle, A_{cap} is the droplet cap area, A_{wet} is the wet base area, and A_{tot} is the total surface area in a unit cell as shown in Fig. 2. During condensation, vapor is condensed on the cap area of the droplets, hence increasing the droplet representative diameter while maintaining the contact angle and the area ratios of Eq. (10) constant. Thus, we can write the rate of latent heat transfer to the surface as [6]

$$\dot{q}''_{latent} = \lambda\dot{m}''\frac{A_{cap}}{A_t} \qquad (11)$$

where $\dot{q}''_{latent}$ is the latent heat rate transferred to the fluid per unit area and λ is the enthalpy of condensation of the vapor at the mean temperature.

Finally, the conservation of energy in the control volume results in

$$\dot{q}''_{surface} + \dot{q}''_{latent} - \dot{q}''_{convection} = 0 \qquad (12)$$

Note that for the case of condensation, $\dot{q}''_{surface}$, q''_{latent}, and $q''_{convection}$ are all defined to be positive. Considering the actual temperature distribution within the micro-drop, the cap of the micro-drop is at the interface saturation temperature, while its bottom is at T_s. The interface saturation temperature, which is the same as the wet-bulb temperature, is lower than the ambient dry-bulb temperature T_∞, but higher than T_s, in order for the condensation process to trigger.

However, the direction of heat transfer due to the phase-change process is independent of temperature gradient. In general, the droplet gives up heat to its surroundings during condensation, and receives heat from it during evaporation. Therefore, although the saturation temperature is less than T_∞, the condensation latent heat is transferred at the wet-bulb temperature from the droplet towards its surroundings.

The heat transfer terms in Eq. (12) are calculated using Eqs. (8), (9), and (11), respectively. Knowing all of the properties and boundary conditions, the only unknown of Eq. (12) is the surface temperature T_s. Equation (12) can be solved by iterative methods to find T_s. The local condensation rate $\dot{m}''$ can then be found for every local position on the surface. The area ratios of Eq. (10) remain almost constant during the self-similar regime, even for small representative base diameters at the early stage of condensation [6]. Thus, a time-stepping process can be implemented in a computer code to calculate the condensation rate at every time step, and to calculate the total condensate mass accumulated on every local position over time.

RESULTS AND DISCUSSION

Humid air is considered as the ambient fluid for numerical evaluation of model results. The values shown in Table 1 are assumed as typical geometrical and material properties for a reference case with reference to Fig. 1.

The contact angle is assumed as shown in Table 1. Its real value is a function of many parameters such as surface properties, liquid properties, temperature, and evaporation (receding) or condensation (advancing) process. The saturation pressures are calculated at each temperature from the correlations of the ASHRAE Standards [14] for humid air. Since laminar flow was assumed for the derivation of the present analytical model, simulation results are applicable for the following range [11]

$$\mathrm{Ra}_x < 10^9\,\mathrm{Pr} \qquad (13)$$

where Pr is the Prandtl number. For conditions where the Rayleigh number is higher than the threshold specified by Eq. (13), the flow will become turbulent and the model results may lose accuracy. For the results discussed here, the Ra_x was checked to meet the criterion of Eq. (13).

Table 1 - Typical geometrical and material properties for model evaluation

Property	Value	Property	Value
g	$9.81\,m/s^2$	λ	$2.27\times10^6\,J/K$
R	$287\,J/kgK$	t	$3\times10^{-3}\,m$
ρ_w	$1000\,kg/m^3$	T_o	$-28\,°C$
k_∞	$0.025\,W/mK$	T_∞	$25\,°C$
k_s	$1\,W/mK$	ϕ_∞	0.6
v	$1.5\times10^{-5}\,m^2/s$	γ	$20°$
α	$2.14\times10^{-5}\,m^2/s$	θ	$90°$
D	$2.5\times10^{-5}\,m^2/s$	L	$1.5\,m$

To validate the model, numerical results were compared with the work of D'Agaro *et al*. [5]. All the geometrical and material properties as mentioned in Table 1 are selected the same as utilized by [5] in order to perform a proper validation of the model.

Figure 3 shows the comparison. The accumulated water condensate thickness is found from

$$H = \sum_{time} \dot{m}'' \bigg/ \rho_w \qquad (14)$$

where ρ_w is the water density and H is the accumulated water condensate thickness. The value of H represents the average local thickness of condensate formed on every position of the surface.

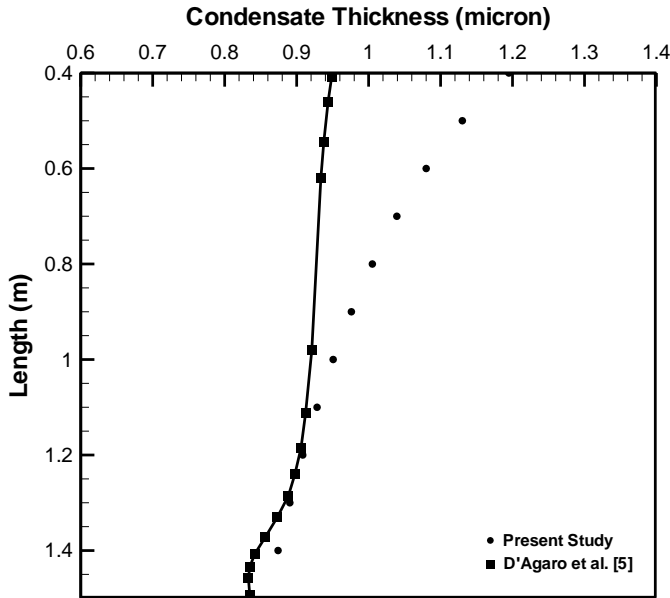**Condensate Thickness (micron)**

Figure 3 - Comparison of accumulated water condensate thickness H with D'Agaro *et al.* [5] after 15 seconds. Refer to Figs. 1 and 2 and Table 1 for geometry and properties.

The present analytical model is developed using the boundary layer theorem which is suitable for local positions that are not too close to the leading edge of the surface. As a result of this assumption, local positions larger than $x = 0.4\,m$ are compared with the work of D'Agaro *et al.*, where relatively good agreement is shown in Fig. 3. In the work of D'Agaro *et al.*, an enclosure is considered for their numerical simulation, and end effects cause the discrepancy between the results in length positions less than $1\,m$.

Figure 4 shows the surface temperature solution at 15 seconds after condensation commencement. The considered glass material has a relatively high thermal conductivity of $k_s = 1\,W/mK$ which causes the temperature at the inner surface side to be close to $-28\,°C$. It is found that the glass is slightly warmer near the leading edge of the surface. Since the rate and accumulation of condensed water is higher for smaller length values, the latent heat is also higher at those positions, which in turn results in slightly higher surface temperatures.
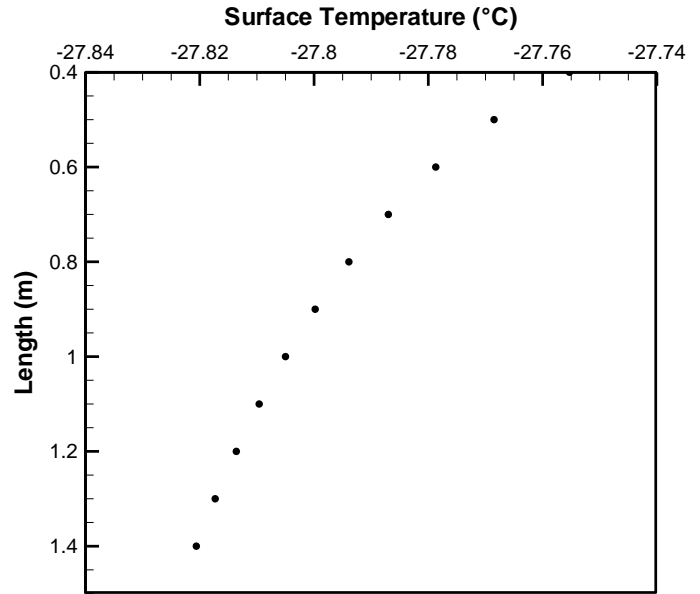
Surface Temperature (°C)

Figure 4 - Surface temperature T_s variation after 15 seconds. Refer to Figs. 1 and 2 and Table 1 for geometry and properties.

As mentioned under the model development section, the solution of vertical flat surface can be generalized to inclination angles of down to $\theta = 30°$. In Fig. 5, the accumulated water condensate thickness is compared for different inclination angles. The results show that by inclination of the surface, the rate of liquid condensation over the surface increases. Figure 6 also shows the temperature variation after 15 seconds of condensation for various inclination angles. Since for higher condensation rates, the latent heat production is higher, the surface temperature is increased by decreasing inclination angle.

Numerical solution of Eq. (12) can be done almost instantly on modern computers and calculators. Although a simple geometrical arrangement was assumed, it is shown that the model can predict the full solution within a reasonable accuracy. Since the CFD simulation of the conjugate flow and heat transfer equations requires more resource and time, the model presented here can be used to blend the analytical natural convection solution with the dew model in order to estimate the mist condensation problem over vertical and inclined surfaces.
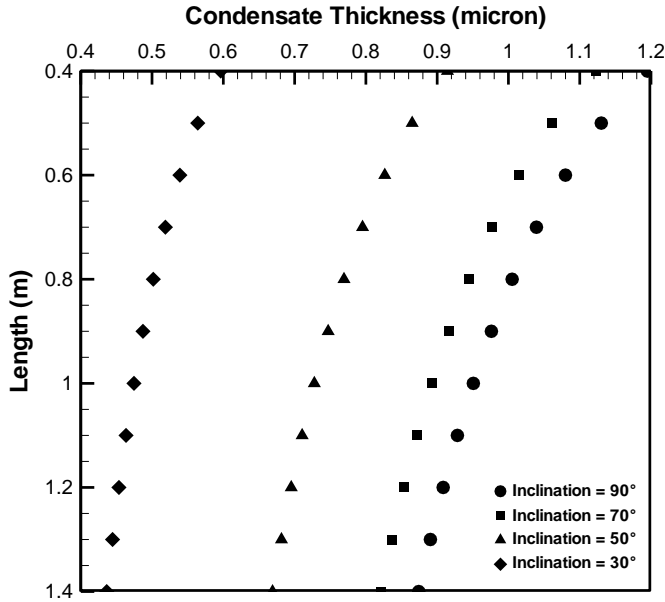
Condensate Thickness (micron)

Figure 5 - Accumulated water condensate thickness H **after 15 seconds for different inclination angles. Refer to Figs. 1 and 2 and Table 1 for geometry and properties.**
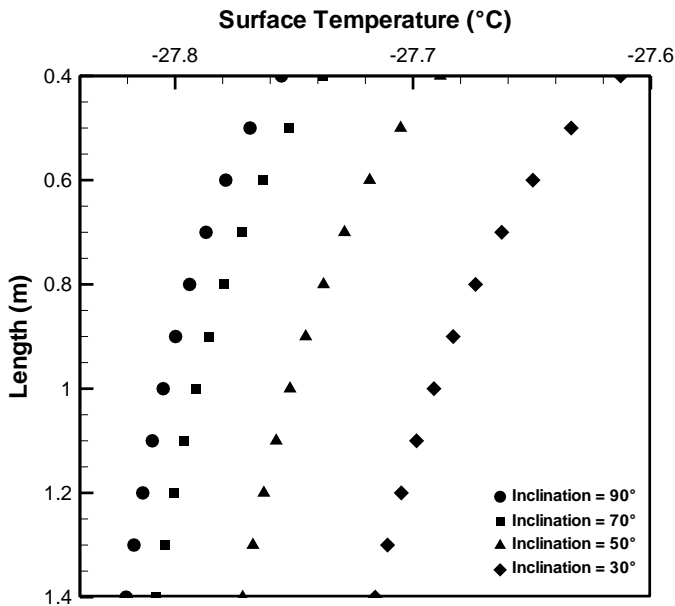
Surface Temperature (°C)

Figure 6 - Surface temperature T_s **variation after 15 seconds for different inclination angles. Refer to Figs. 1 and 2 and Table 1 for geometry and properties.**

CONCLUSIONS

The problem of mist condensation over an inclined vertical surface is studied in this paper. The conjugate fluid flow and heat transfer equations, as well as the heat transfer equation across the solid surface are solved analytically. A dew formation model is used to predict the shape and size of the droplets. Due to the analytical approach considered for solving the natural convective heat and mass transfer, fully numerical calculations are avoided for solving the equations.

It was shown that although a simple geometry is considered, the model can predict the bulk of the mist formation with reasonable accuracy. The model can be used for estimating the formation of dew droplets over surfaces exposed to mist condensation, especially where the fogging needs to be prevented by appropriate defogging apparatus.

NOMENCLATURE

A_{cap}	Unit Cell Spherical Cap Area (m^2)
A_{tot}	Unit Cell Total Base Area (m^2)
A_{wet}	Unit Cell Wet Base Area (m^2)
D	Mass Diffusivity (m^2/s)
g	Gravitational Acceleration (m/s^2)
Gr	Grashof Number ($g\beta x^3 \Delta T / v^2$)
h	Convection Coefficient ($W/m^2 K$)
H	Accumulated Water Condensate Thickness (m)
k_∞	Ambient Fluid Thermal Conductivity (W/mK)
k_s	Surface Thermal Conductivity (W/mK)
Nu_x	Nusselt Number (hx/k)
Pr	Prandtl Number (v/α)
Ra_x	Rayleigh Number ($g\beta x^3 \Delta T / v\alpha$)
T_m	Mean Temperature $(T_\infty + T_s)/2$ (K)
T_o	Outside Temperature (K)
T_∞	Ambient Fluid Temperature (K)
T_s	Surface Temperature (K)
$\dot{m}''$	Mass Entrainment Rate Per Unit Area ($kg/m^2 s$)
p	Vapor Saturation Pressure (Pa)
$\ddot{q}''_{convection}$	Convection Rate Per Unit Area (W/m^2)
$\ddot{q}''_{surface}$	Conduction Rate Per Unit Area (W/m^2)
$\ddot{q}''_{latent}$	Latent Heat Rate Per Unit Area (W/m^2)
R	Specific Gas Constant (J/kgK)
Sc	Schmidt Number (v/D)
Sh_x	Sherwood Number (Kx/D)
t	Surface Thickness (m)
x	Vertical Position (m)

Greek Letters

α	Thermal Diffusivity (m^2/s)
β	Volumetric Thermal Expansion Coefficient ($1/K$)
δ	Boundary Layer Thickness (m)
ϕ	Relative Humidity
γ	Contact Angle
λ	Enthalpy of Condensation (J/kg)

ν Kinematic Viscosity (m^2/s)

θ Inclination Angle ($^\circ$)

ρ Vapor Density (kg/m^3)

ρ_w Water Density (kg/m^3)

Subscripts
s Surface
∞ Ambient
o Outside

REFERENCES

[1] Merte, H., & Yamali, C. (1983). Profile and departure size of condensation drops on vertical surfaces. Heat and Mass Transfer, 180, 171–180.

[2] Aroussi, A., Hassan, A., & Morsi, Y. S. (2003). Numerical simulation of the airflow over and heat transfer through a vehicle windshield defrosting and demisting system, Heat and Mass Transfer, 39, 401–405. doi:10.1007/s00231-002-0307-x

[3] Croce, G., Beaugendre, H., & Habashi, W. G. (2002). Numerical simulation of heat transfer in mist flow. Numerical Heat Transfer, 42(2002), 139–152. doi:10.1080=10407780290059477

[4] Croce, G., D'Agaro, P., De Angelis, A., & Mattiello, F. (2007). Numerical simulation of windshield defogging process. Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, 221(10), 1241–1250. doi:10.1243/09544070JAUTO132

[5] D'Agaro, P., Croce, G., & Cortella, G. (2006). Numerical simulation of glass doors fogging and defogging in refrigerated display cabinets. Applied Thermal Engineering, 26(16), 1927–1934. doi:10.1016/j.applthermaleng.2006.01.014

[6] Croce, G., De Candido, E., & D'Agaro, P. (2009). Numerical modeling of heat and mass transfer in finned dehumidifier. Applied Thermal Engineering, 29(7), 1366–1374. doi:10.1016/j.applthermaleng.2008.05.003

[7] Croce, G., D'Agaro, P., & Della Mora, F. (2005). Numerical simulation of glass fogging and defogging. International Journal of Computational Fluid Dynamics, 19(6), 437–445. doi:10.1080/10618560500233479

[8] Beysens, D. (1995). The formation of dew. Atmospheric Research, 39(1-3), 215–237. doi:10.1016/0169-8095(95)00015-J

[9] Beysens, D. (2006). Dew nucleation and growth. ComptesRendus Physique, 7(9-10), 1082–1100. doi:10.1016/j.crhy.2006.10.020

[10] Lienhard, J. H. IV, Lienhard, J. H. V, 2004, A Heat Transfer Textbook, Phlogiston Press.

[11] Bejan, A., Convection Heat Transfer, John Wiley & Sons Inc., 2004.

[12] Lin, H., & Wu, C. (1995). Combined heat and mass transfer by laminar natural convection from a vertical plate. Heat and mass transfer, 30(6), 369–376. doi:10.1007/BF01647440

[13] De Candido, E., Croce, G., & D'Agaro, P. (2012). Droplet Buildup and Water Retention Prediction in Condensation Processes. Heat Transfer Engineering, 33(13), 1130–1137. doi:10.1080/01457632.2012.663687

[14] ASHRAE Handbook of Fundamental, American Society of Heating, Refrigerating, and Air Conditioning, Atlanta, GA, 1988.